

Casimir expulsion of periodic configurations

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There is the possibility in principle that the noncompensated Casimir force exists in open nano-sized metal cavities arranged in the form of periodic structures. It is found that when trapezoid cavities are strictly periodic all the Casimir expulsion forces are completely compensated. However, when the distance of the gap is formed between the cavities, in the periodic configuration a noncompensated expulsion force proportional to the number of cavities appears. There are such effective parameters of the periodic configuration (the angles of the opening of cavities, their lengths and the relationships between them) which lead to the appearance of a maximum of expulsion forces per unit of structure length.

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In Ref. [1] the possibility in principle is shown that the noncompensated Casimir force can exist in open nano-sized metal cavities. The effect is theoretically demonstrated for a single trapezoid configuration. The force manifests itself as the time-constant expulsion of open cavities in the direction of their least opening. The optimal parameters of the angles of opening (broadening) of the cavities' generatrices and their lengths are found, at which the expulsion force is maximal. It should be noted that the force differs significantly from expulsion forces capable of creating effects of levitation-type over bodies-partners [2–6]. The question arises if the existence of noncompensated expulsion forces is possible in periodic structures based on trapezoid configurations possessing the effect of expulsion. A particular case of trapezoid configurations is Casimir parallel mirrors [7, 8] which do not possess an effective expulsion force [1].

Let us consider a periodic configuration with trapezoid cavities as an illustration of the possibility of the Casimir expulsion force existence. Note that a single cavity is understood as an open thin-walled metal shell with one or several outlets. The inner and outer surfaces of the cavity should have the properties of perfect mirrors. The cavity should entirely be immersed into a material medium or be a part of the medium with the parameters of dielectric permeability being different from those of physical vacuum. In Cartesian coordinates the configuration looks like two thin metal plates with the surface width L (oriented along the z -axis) and length R , which are situated at a distance a from one another; the angle 2φ of the opening of the generating lines of cavities between the plates can be varied (by the same value φ imultaneously for both wings of the trapezoid cavity) as it is shown in Fig.1.

The periodic configuration with trapezoid figures in the Cartesian coordinates looks like it is shown in Fig. 2. Each figure in the configuration period is similar to a single figure in Fig. 1. Between the ends of the figures in the period there is a distance of the gap d .

For each figure the expulsion force in the x -direction

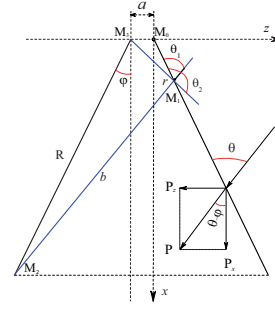


FIG. 1. Schematic view of the configuration of a symmetric trapezoid cavity with the length R of the wing surface, a particular case of which is parallel planes, i.e. $\varphi = 0$, and a triangle at $a = 0$. The section of the cavity shown in the Cartesian coordinates in the plane (x, z) has the width L in the y direction normal to the plane of the figure. The blue straight lines designate virtual rays with the length b coming from the point M_1 at limit angles Θ_1 and Θ_2 onto the right cavity surface ending at the ends of the opposite cavity wing at the points M_2 and M_3 , respectively.

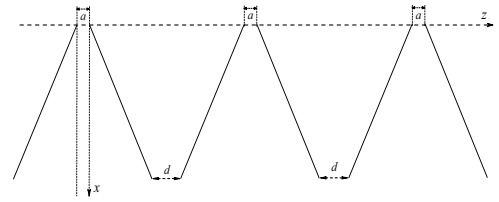


FIG. 2. Schematic view of the periodic configuration, in which the broadened parts of the symmetrical trapezoid cavities with the distance of the gap d between them are in the x -direction.

can be found in the first approximation in the form [1]

$$F_x = \int_0^L dy \int_0^R P_x(\varphi, \Theta, r) dr. \quad (1)$$

Here, the local specific force of expulsion at each point r

on the cavity wing with the length R and width L is

$$P_x(r) = \frac{\hbar c \pi^2}{240 s^4} \int_{\Theta_1}^{\Theta_2} \sin(\Theta - 2\varphi)^4 \cos(\Theta - \varphi) d\Theta \quad (2)$$

$$= -\frac{\hbar c \pi^2}{240 s^4} A(\varphi, \Theta_1, \Theta_2),$$

where

$$A(\varphi, \Theta_1, \Theta_2) = \frac{1}{240} \left[90 \sin(\varphi - \Theta_1) - 90 \sin(\varphi - \Theta_2) \right. \\ + 60 \sin(3\varphi - \Theta_2) - 60 \sin(3\varphi - \Theta_1) \\ + 20 \sin(5\varphi - 3\Theta_2) - 20 \sin(5\varphi - 3\Theta_1) \\ + 5 \sin(7\varphi - 3\Theta_1) - 5 \sin(7\varphi - 3\Theta_2) \\ \left. + 3 \sin(9\varphi - 5\Theta_1) - 3 \sin(9\varphi - 5\Theta_2) \right]. \quad (3)$$

In formula (2), $\hbar = h/2\pi$ is reduced Planck constant, c is light velocity, and the functional expressions for limit angles Θ_1 , Θ_2 (see Fig. 1) in the trapezoid cavity and the parameter s are

$$\Theta_1 = \arccos \left\{ -(r + a \sin \varphi - R \cos 2\varphi) \right. \\ \times \left[(a + R \sin \varphi + r \sin \varphi)^2 \right. \\ \left. + (r \cos \varphi - R \cos \varphi)^2 \right]^{-\frac{1}{2}} \Big\}, \quad (4)$$

$$\Theta_2 = \arccos \left[-\frac{r + a \sin \varphi}{\sqrt{a^2 + r^2 + 2ra \sin \varphi}} \right], \quad (5)$$

and

$$s = \frac{\sin(2\varphi - \Theta_2)(a + r \sin \varphi)}{\sin(\varphi - \Theta_2)}. \quad (6)$$

When such cavities are being arranged in the periodic structure, the following should be kept in mind. The periodic arrangement of n trapezoid cavities being at the distance d from one another, which sides with the widest opening are directed against the x axis, leads to the formation of $n-1$ cavities with oppositely directed openings (see Fig. 2). In this case, a wing (one of the surfaces of the trapezoid cavity) of each cavity is a wing of the other cavity, the opening of which is oppositely directed. Thus, for n cavities periodically arranged along the y axis we can write the expression for the total force of expulsion along the x axis

$$F_\Sigma = nF_x(a) - (n-1)F_x(d). \quad (7)$$

Here, $F_x(a)$ is the force along the x axis for the distance a between the nearest ends of the cavities, and $F_x(d)$

is, respectively, the force for the distance of the gap d between periods instead of a in formulas (1-6). From formula (7) it is clear that for $d = a$ in the strictly periodic configurations the expulsion force is $F_\Sigma \rightarrow F_x(a)$ at $n \rightarrow \infty$. That is even at $n \rightarrow \infty$ the thrust force in a strictly periodic structure always remains at the level of expulsion forces for a single cavity and is directed to the least opening of the cavity wings. It means that in the configuration the thrust force will be created which is directed against the x -direction. However, it is clear that at $d \neq a$ the expulsion force of the periodic configuration will not remain at the same level and will depend on the d/a relation according to formula (7) for different angles φ of the opening of cavities as it is shown in Fig.3(a). When the number n of trapezoid cavities in the periodic configuration is growing, the character of the curves will be similar to that of the curves presented; however, of course, their level along the coordinate F_x for any angles φ and parameters will grow linearly.

It is possible to determine the effectiveness Q of the expulsion of n cavities as the relation of the total force F_Σ to the entire length of the configuration along the y axis

$$Q = \frac{F_\Sigma}{n(a + 2R \tan \varphi) + (n-1)d}. \quad (8)$$

The dependence of Q on the d/a relation is displayed in Fig.3(b). It can be seen, for example, that at $a = 4 \times 10^{-9}$ m, for any length of the cavity wings R with different angles φ , there is a maximum of effectiveness Q of expulsion. As is known [1] there is a maximum of the expulsion forces for each trapezoid figure depending on the angle of the opening of the cavities' wings [Fig. 3(c)] and their lengths. In the periodic configurations with the distance of the gap d , there is a maximum of the expulsion effectiveness Q as well. The maximum of effectiveness, which is common for two parameters φ and d/a , is shown in Fig. 4. It was found that for $a = 4 \times 10^{-9}$ m and $R/a = 2.5$, the best angle is $\varphi \approx 5.59$ deg at $d/a \approx 1.58$ and $n = 2$. For the given relation R/a at $n \rightarrow \infty$ the angle is $\varphi \rightarrow 6.0$ deg and $d/a \rightarrow 1.7$. At $R/a \rightarrow 20$ and $n \rightarrow \infty$, the angle $\varphi \rightarrow 0.1$ deg and $d/a \rightarrow 1.85$.

Note that when the trapezoid cavities are chequerwisely arranged in the periodic configuration, i.e. the cavity with the widening of the opening against the x axis is put to the cavity with the wide opening in the x -direction (so that the boundary wings become common to both cavities), in the system there will be a torque moment around the centre of mass of the configuration even at $d = a$. Of course, at certain combinations of the arrangement of cavities in the periodic configuration, a much larger torque can be achieved at $d \neq a$ compared to that at $d = a$.

Thus, in the present paper, the possibility in principle is shown that the noncompensated Casimir force exists in open nanosized metal cavities arranged in the form

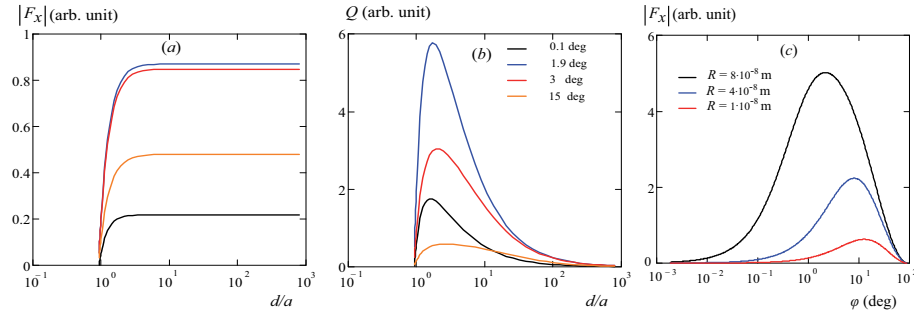


FIG. 3. Absolute values of the Casimir expulsion forces in the periodic configuration in the direction opposite to the x -axis depending on the relation d/a in the structure (a) and the effectiveness Q of expulsion of the structure (b) at different angles φ of the opening of the cavities. The total Casimir force of expulsion (c) for different lengths R depending on the angle φ .

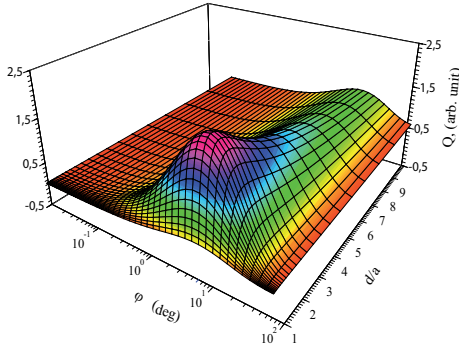


FIG. 4. Effectiveness Q of the Casimir expulsion of the periodic structure depending on the angle φ and relation d/a .

of periodic structures. It is found that in strictly periodic structures based on trapezoid figures all the Casimir expulsion forces are practically completely compensated. However, when the distance of the gap is formed between the cavities, in the periodic configuration a noncompensated expulsion force appears. In this case, at any rela-

tions of the configuration parameters (angles of opening and wing length of the cavities, the distance between the cavities, etc.) and at any number of cavities in the period there is an effective maximum of the expulsion forces. In some periodic structures there can be a torque moment around the centre of mass of the configuration.

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